Parametric Identification of a Nonlinear System Using Multi-harmonic Excitation

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Abstract

Parametric identification of a single degree-of-freedom (SDOF) nonlinear Duffing oscillator is carried out using a harmonic balance (HB) method. The parameters of the system are obtained using a harmonic input, for the case of periodic response. Problems of matrix inversion, due to poor conditioning are sometimes encountered in the computation. This may occur due to large differences in the relative values of inertia, damping and spring forces or dependence of these parameters on one another. The inversion problem may also occur due to a poor choice of the excitation signal frequency and amplitude. However there is limited choice of adjustable input parameters in this case. In this work, an extended HB method, which uses a combination of two harmonic inputs, is suggested to overcome the above problem.

1 Introduction

System identification involves obtaining the structure and the unknown parameters of the model of a system from the input-output information. If the mathematical structure of the governing equations of the system is assumed or known a priori, the problem of identification reduces to the determination of unknown parameters of the system. This problem is termed as parametric identification. If the structure of the equations is also unknown, the problem is called a non-parametric identification. The latter is obviously a more difficult problem than the former. The system may be viewed as an operator or a function that maps the input signal to the output signal. For the identification, the input-output information for a real system is required to be available from suitable tests conducted on the system. The successful identification method will yield a model with parameters such that the input used in the experiment if given to the identified model would give the same output as that of the original system. Thus with the identified system, one can obtain the response for any arbitrary excitation. Also, the output of the system at a future time can be predicted.

The theory of linear system identification is well developed. In many applications the systems are nonlinear or the effect of nonlinearity cannot be ignored. In such cases nonlinear system identification has to be carried out. The presence of nonlinearities poses challenges. There are a wide variety of methods available for nonlinear system identification.[1] In the present work the area of investigation is confined to the parametric identification of a SDOF system.

In one of early classical papers on nonlinear system identification, Masri and Caughey[1] used the state variables of nonlinear systems to express the system characteristics in terms of orthogonal functions.

Various investigations[2–5] have been done in the area of nonlinear vibratory system identification. Yasuda *et al.*[6] applied the principle of harmonic balance for the identification of nonlinear multi degree-of-freedom systems. They approximated the nonlinearity in the system using polynomials and the response of the system was represented by a truncated Fourier series. Yuan and Feeny[7] use the technique of combining the periodic orbit extraction and harmonic balance scheme for the nonlinear parametric identification of chaotic systems.

In using HB for system identification, numerical difficulties are often encountered. The main objective of this paper is to develop a robust identification procedure using HB technique. This is based on a multi-harmonic excitation, which is shown to be better at parametric identification for a broad range of excitation frequencies and amplitudes.

2 Solution of Nonlinear System by Harmonic Balance Method

Let us consider vibratory systems governed by nonlinear ordinary differential equations and subjected to periodic force excitation. Further, one can assume that the response of the system is periodic. In the Harmonic balance (HB) method, the periodic response of the system is expressed in terms of a truncated Fourier series with terms having frequencies, which are integer multiples/submultiples of the excitation frequency. The coefficients of the harmonic terms are adjusted systematically by an iterative procedure such that the resulting time series matches with the original one to the required degree of accuracy. The original response of the system in general cannot be represented in an analytical form, as the system is nonlinear. The HB solution thus obtained can be used for the system analysis as well as system identification.

In the identification scheme, the HB solution is substituted into the governing equation of motion, which gives an algebraic equation in terms of system parameters. Imposing the condition that this equation should be satisfied at all sample points, yields a system of linear algebraic equations. This is solved by a psuedo-inversion to obtain the unknown system parameters. Consider a Duffing oscillator, which is periodically forced with L term harmonic excitation, given by the equation

$$m\ddot{x} + c\dot{x} + k_l x + \alpha x^3 = f_0 + \sum_{l=1}^{L} f_l \cos(l\Omega t) + g_l \sin(l\Omega t) \tag{1}$$

Assume that the response $x(t)$ of a vibrating system is available. Let it be periodic of fundamental period T. Expressing the response in a truncated M term harmonic Fourier series where $M > L$, we have

$$x(t) = a_0 + \sum_{k=1}^{M} (a_k \cos k\Omega t + b_k \sin k\Omega t) \tag{2}$$

The following steps are involved in HB method

1. Divide the time period of the response into N number of equal intervals of size $dt = T/N$. Choose sampling rate to satisfy the Nyquist criterion.
2. Evaluate $x(t)$ at these times $x(0), x(\Delta t), x(2\Delta t), \ldots, x((N-1)\Delta t)$ for a period of the response signal.

3. For M harmonic terms this can be written as

$$\begin{bmatrix} x(0) \\ x(\Delta t) \\ x(\Delta t) \\ \cdot \\ \cdot \\ x((N-1)\Delta t) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & \cdot & 1 & 0 \\ 1 & \cos(\Omega\Delta t) & \sin(\Omega\Delta t) & \cdot & \cos(M\Omega\Delta t) & \sin(M\Omega\Delta t) \\ 1 & \cos(2\Omega\Delta t) & \sin(2\Omega\Delta t) & \cdot & \cos(M\Omega 2\Delta t) & \sin(M\Omega 2\Delta t) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cos(\Omega(N-1)\Delta t) & \sin(\Omega(N-1)\Delta t) & \cdot & \cos(M\Omega(N-1)\Delta t) & \sin(M\Omega(N-1)\Delta t) \end{bmatrix}$$

$$\times \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ \cdot \\ a_M \\ b_M \end{bmatrix} \tag{3}$$

In a compact form the above equation is $X = \Gamma A$. Assume 'A' initially and evaluate X.

4. Evaluate $h(x) = x^3$, the nonlinear terms for the known values of x. The nonlinear function is evaluated in time domain as

$$H = \{h(x(t))\} = \begin{bmatrix} h(0) \\ h(\Delta t) \\ \cdot \\ \cdot \\ h((N-1)\Delta t) \end{bmatrix} \tag{4}$$

Express $h(x)$ in time domain in Fourier series as

$$h(x(t)) = q_0 + \sum_{k=1}^{M} q_k \cos k\Omega t + r_k \sin k\Omega\, t \tag{5}$$

In compact form, $H = \Gamma Q$

H can be obtained from $X(h(x) = \alpha x^3$ for each term). Thus

$$Q = F(H) = F(H(X)) = F(a_0, a_1, b_1, \ldots, a_M, b_M) \tag{6}$$

Using $\omega_n = \sqrt{k_l/m}$ and $\xi = c/(2m\omega_n)$, the complete equation has the following form

$$\begin{bmatrix} \omega_n^2 & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \omega_n^2 - \Omega^2 & 2\xi\omega_n\Omega & \cdot & 0 & 0 \\ 0 & -2\xi\omega_n\Omega & \omega_n^2 - \Omega^2 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & \cdot & \omega_n^2 - (M\Omega)^2 & 2\omega_n\xi M\Omega \\ 0 & 0 & 0 & \cdot & -2\omega_n\xi M\Omega & \omega_n^2 - (M\Omega)^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ \cdot \\ a_M \\ b_M \end{bmatrix} + \begin{bmatrix} q_0 \\ q_1 \\ r_1 \\ \cdot \\ q_M \\ r_M \end{bmatrix} - \begin{bmatrix} f_0 \\ f_1 \\ g_1 \\ \cdot \\ f_L \\ g_L \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

$$YA + Q - F_e = 0 \tag{8}$$

where Y is Jacobian matrix of the linearised form of the equation. Note that the above equation is a function of the coefficients $a_0, a_1, b_1, \ldots, a_M$ and b_M.

$$\text{i.e. } f_i(a_0, a_1, b_1, \ldots, a_M, b_M) = 0 \text{ for } i = 1, 2, \ldots, 2M + 1 \tag{9}$$

The basic objective is to obtain the coefficients so as to satisfy the above equations.

2.1 Newton–Raphson method

Solution of the above set of simultaneous nonlinear algebraic equations 2.9 can be accomplished by Newton-Raphson (NR) method. An initial set of Fourier coefficients $a_00, a_10, b_10, \ldots, a_{M0}, b_M0$ is assumed. Fourier series coefficients obtained from the solution of the corresponding linear system are taken as the initial guess. Using Taylor series expansion about this point,

$$\begin{aligned} f_i(a_0, a_1, b_1, \ldots, a_M, b_M) &= f_i(a_{00}, a_{10}, b_{10}, \ldots, a_{M0}, b_{M0}) \\ &+ \left(\frac{\partial f_i}{\partial a_0}\right)_{a_{00}, a_{10}, b_{10}, \ldots, a_{M0}, b_{M0}} \Delta a_0 + \left(\frac{\partial f_i}{\partial a_1}\right)_{a_{00}, a_{10}, \ldots} \Delta a_1 + \cdots + \left(\frac{\partial f_i}{\partial b_M}\right)_{a_{00}, a_{10}, \ldots} \Delta b_M \\ &+ \text{Higher order terms } i = 1, 2, \ldots, 2M + 1 \end{aligned} \tag{10}$$

where $a_0 = a_{00} + \Delta a_0$; $a_1 = a_{10} + \Delta a_1$; $\ldots$; $b_M = b_{M0}^{+} \Delta b_M$.

Let $nt = 2M + 1$. Neglecting higher order terms we have

$$\begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ f_{nt} \end{bmatrix}_i = \begin{bmatrix} f_{10} \\ f_{20} \\ \cdot \\ \cdot \\ f_{nt0} \end{bmatrix}_i + \begin{bmatrix} \frac{\partial f_1}{\partial a_0} & \frac{\partial f_1}{\partial a_1} & \frac{\partial f_1}{\partial b_1} & \cdot & \frac{\partial f_1}{\partial b_M} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_{nt}}{\partial a_0} & \frac{\partial f_{nt}}{\partial a_1} & \frac{\partial f_{nt}}{\partial b_1} & \cdot & \frac{\partial f_{nt}}{\partial b_M} \end{bmatrix}_i \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \\ \cdot \\ \cdot \\ \Delta b_M \end{bmatrix}_i \tag{11}$$

This can be written as $F_i = F_{0i} + J_i \Delta A_i$

$$J = \frac{\partial F}{\partial A} \; J_{ij} = \frac{\partial F_i}{\partial A_j} \; i, j = 1, 2, \ldots, nt \tag{12}$$

The net force on the system is the sum of linear forces YA_i, nonlinear forces Q_i and the external forces F_{ei}. Y is the Jacobian matrix of the linear part and suffix 'i' denotes the iteration number. The net force at the end of i^{th} iteration is

$$F_i = YA_i + Q_i - F_{ei} \tag{13}$$

The total Jacobian matrix J is given by

$$J = \frac{\partial F}{\partial A} = Y + \frac{\partial Q}{\partial A} Q = (\Gamma^T \Gamma)^{-1} \Gamma^T H \tag{14}$$

$$\frac{\partial Q}{\partial A} = (\Gamma^T \Gamma)^{-1} \Gamma^T \frac{\partial H}{\partial A} \frac{\partial H}{\partial A} = \frac{\partial H}{\partial X} \frac{\partial X}{\partial A} \tag{15}$$

Using $X = \Gamma A$

$$\frac{\partial Q}{\partial A} = (\Gamma^T \Gamma)^{-1} \Gamma^T \begin{bmatrix} h'(0) & 0 & \cdot & 0 & 0 \\ 0 & h'(\Delta t) & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & 0 & h'(T - \Delta t) \end{bmatrix} \Gamma \tag{16}$$

The total Jacobian matrix J is evaluated at every step as

$$J = Y + \frac{\partial Q}{\partial A} = Y + (\Gamma^T \Gamma)^{-1} \Gamma^T \operatorname{diag}(H') \Gamma \tag{17}$$

Since the Jacobian matrix for the linear part Y is constant, it is to be evaluated only once for the purpose of computation.

The N-R algorithm can be summarised as follows.

(i) Assume the initial Fourier coefficient set A. Solve for ΔA_i which makes $F_i = 0$ in Eq. (2.12) i.e., $\Delta A_i = -J_i^{-1} F_{0i}$
(ii) Set the new values for coefficients in iteration as $A_{i+1} = A_i + \Delta A_i$
(iii) Calculate F_i and check if $\|F_i\|_\infty <$ tolerance, where $\|F_i\|_\infty := \max_i |F_i|$
(iv) If the above condition is satisfied the computation stops. Otherwise repeat the above process until the required tolerance condition is satisfied.The final set A gives the coefficients of periodic solution.

3 Parametric Identification Using Harmonic Balance Method

Consider the Duffing oscillator given by the equation

$$m\ddot{x} + c\dot{x} + k_l x + \alpha x^3 = f_e(t) \tag{18}$$

where

$$f_e(t) = f_0 + \sum_{l=1}^{L} f_l \cos(l\Omega t) + g_l \sin(l\Omega t) \tag{19}$$

where m, c, k_l and α are the parameters of the system to be determined from the known input-response data. For identification purpose, the oscillator is assumed to be excited harmonically with known excitation parameters. Assuming the solution to be periodic, this can be expressed as in equation 2.2. Substituting this periodic solution in (18), we get

$$
\begin{aligned}
&- m\sum_{k=1}^{M}(a_k k^2\Omega^2\cos k\Omega t + b_k k^2\Omega^2 \sin k\Omega t) + c\sum_{k=1}^{M}(-a_k k\Omega\sin k\Omega t + b_k k\Omega\cos k\Omega t) \\
&+ k_l\left(a_0 + \sum_{k=1}^{M}(a_k\cos k\Omega t + b_k\sin k\Omega t)\right) + \alpha\left(a_0 + \sum_{k=1}^{M}(a_k\cos k\Omega t + b_k\sin k\Omega t)^3\right) = f_e(t)
\end{aligned}
\tag{20}
$$

Let

$$
\begin{aligned}
&-\Omega^2\sum_{k=1}^{M}k^2(a_k\cos k\Omega t + b_k\sin k\Omega t) = p_1(t);\quad \Omega\sum_{k=1}^{M}k(-a_k\sin k\Omega t + b_k\cos k\Omega t) = p_2(t); \\
&a_0 + \sum_{k=1}^{M}(a_k\cos k\Omega t + b_k\sin k\Omega t) = p_3(t);\quad \left(a_0 + \sum_{k=1}^{M}(a_k\cos k\Omega t + b_k\sin k\Omega t)\right)^3 \\
&\qquad = p_4(t) \text{ and } f_e = p_5(t)
\end{aligned}
\tag{21}
$$

Thus the (20) can be written as

$$
mp_1(t) + cp_2(t) + k_l p_3(t) + \alpha p_4(t) = p_5(t) \tag{22}
$$

For a set of N discrete times, the matrix form of the equation is

$$
\begin{bmatrix}
p_1(0) & p_2(0) & p_3(0) & p_4(0) \\
p_1(\Delta t) & p_2(\Delta T) & p_3(\Delta T) & p_4(\Delta T) \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
p_1((N-1)\Delta t) & p_2((N-1)\Delta t) & p_3((N-1)\Delta t) & p_4((N-1)\Delta t)
\end{bmatrix}
\begin{Bmatrix} m \\ c \\ k_l \\ \alpha \end{Bmatrix}
= \begin{Bmatrix} p_5(0) \\ p_5(\Delta t) \\ \cdot \\ \cdot \\ p_5((N-1)\Delta t) \end{Bmatrix}
\tag{23}
$$

This can be written as $Gr = f$ The parameters 'r' are obtained from the equation

$$
r = (G^T G)^{-1} G^T f = CL^{-1} G^T f \tag{24}
$$

where

$$CL = G^T G = \begin{bmatrix} p_1(0) & \cdot & \cdot & p_1((N-1)\Delta t) \\ p_2(0) & \cdot & \cdot & p_2((N-1)\Delta t) \\ p_3(0) & \cdot & \cdot & p_3((N-1)\Delta t) \\ p_4(0) & \cdot & \cdot & p_4((N-1)\Delta t) \end{bmatrix}$$

$$\times \begin{bmatrix} p_1(0) & p_2(0) & p_3(0) & p_4(0) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ p_1((N-1)\Delta t) & p_2((N-1)\Delta t) & p_3((N-1)\Delta t) & p_4((N-1)\Delta t) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=0}^{N-1} (p_1(i\Delta t))^2 & \sum p_1(i\Delta t)p_2(i\Delta t) & \sum p_1(i\Delta t)p_3(i\Delta t) & \sum p_1(i\Delta t)p_4(i\Delta t) \\ \cdot & \sum (p_2(i\Delta t))^2 & \sum p_2(i\Delta t)p_3(i\Delta t) & \sum p_2(i\Delta t)p_4(i\Delta t) \\ \cdot & \cdot & \sum (p_3(i\Delta t))^2 & \sum p_3(i\Delta t)p_4(i\Delta t) \\ \cdot & \cdot & \cdot & \sum (p_4(i\Delta t))^2 \end{bmatrix} \tag{25}$$

3.1 Identification: Single harmonic excitation versus two-harmonic excitation

Invertibility of the CL matrix plays a crucial role in successful identification. While it is clear that the excitation parameters $f_1, f_2, \ldots, f_L$; $\Omega_1, \Omega_2, \ldots,$ and Ω_L will influence the relative values of the elements of the CL matrix, the right choice of them a priori is a difficult problem. In this work a two-harmonic input is considered for the identification and a comparative study on the success of identification is carried out. Consider the system subjected to two harmonic excitations.

$$m\ddot{x} + c\dot{x} + k_l x + \alpha x^3 = f_1 \cos \Omega_1 t + f_2 \cos \Omega_2 t \tag{26}$$

A special case where Ω_2 is an integer multiple of $\Omega_1 (\Omega_2 = 2\Omega_1)$ is considered i.e., Ω_2/Ω_1 is rational.

3.2 Implementation

If the periodic response of system is given, an approximate solution for this can be accomplished by performing a Discrete Fourier Transform (DFT) on the response data. The data points should be equally spaced in time. In practice a Fast Fourier Transform (FFT) is used. The FFT coefficients, which are complex, may be converted into equivalent real valued Fourier coefficients. Following this, the system identification by pseudo inverse as mentioned above can be done.

Parametric identification of a Duffing oscillator was carried out using the harmonic balance method. To generate data for the study, the periodic response of a known Duffing oscillator to a harmonic excitation was obtained by simulation in MATLAB. The above data is then used in the harmonic balance method to obtain the Fourier series solution to the problem. Using the above input-output data again, the inverse form of the harmonic balance method is made use of to identify the parameters of the system. The result obtained for a case with single harmonic excitation is shown in Table 1 and Fig. 1. The number of sampling points N considered in this case is 128. Thus the equally spaced sampling time interval is $\Delta t = T/N$ where T is the steady state period of response. Relatively insignificant Fourier coefficient terms are not included in the analysis.

Table 1 Comparison of original and identified parameters

Excitation parameters: $f_1 = 2, \Omega = 0.4$	
Original system parameters: $[m, c, k_l, \alpha]$	Corresponding identified parameters
1.0, 0.2, 1, 1	1.0000, 0.1994, 0.9988, 1.0010

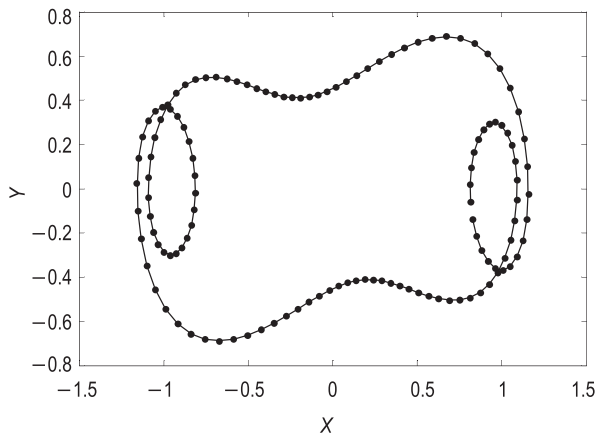

Fig. 1 Phase plane plot of the original and the identified system. Solid line is the one-period steady state response of the original system. The dot symbols stands for the corresponding identified system

3.3 Direct determination of error in parameters

If the identification is carried out using the simulated data as mentioned earlier, the parameters of the original and the identified system are available for error estimation. Error measure based on difference in parameter values can be defined as

$$m_{er} = (m_o - m_i)/m_o;\ c_{er} = (c_o - c_i)/c_o;\ k_{\mathrm{ler}} = (k_{lo} - k_{li})/k_{lo};\ \alpha_{er} = (\alpha_o - \alpha_i)/\alpha_o \tag{27}$$

where o-original i-identified, m, c, k_l and α are the mass, damping coefficient, linear stiffness and nonlinear stiffness respectively; m_{er}, c_{er}, k_{ler} and α_{er} are the error in the identification of mass, damping, linear stiffness and nonlinear stiffness respectively. Total parameter error is computed as

$$E_{\mathrm{par}} = \sqrt{(m_{er}^2 + c_{er}^2 + k_{\mathrm{ler}}^2 + \alpha_{er}^2)/4} \tag{28}$$

4 Results

A computational test to illustrate the improvement of results for two-harmonic input over single harmonic is given here. For comparison of these two cases, the amplitudes of excitation are chosen in the following way. For any F and Ω we set the following: Case 1: Single harmonic excitation amplitude $f_1 = 1.2F$.

Table 2 Comparison of Single and two harmonic excitation: Sweeping across F-Ω range

Original Parameter set $[mck_l\alpha]$ = [0.1 0.02 10.1]; Frequency ratio $\eta = \Omega/\omega_n$, ω_n = undamped natural frequency of the corresponding linear system F: 0.1 to 2.1, Step size = 0.1; η = 0.1 to 0.5, Step size = 0.02, Number of trials NT = 441(21 × 21)			
$E1$: Number of cases with $E_{par} \leq 1\%$; $E2$: E_{par} 1 to 5%; NP: Not periodic; Net Success = $(E1 + E2)/(NT - NP)$			
	E1	E2	Net success%
One harmonic input	233	84	71.8821
Two harmonic input	358	58	94.3311

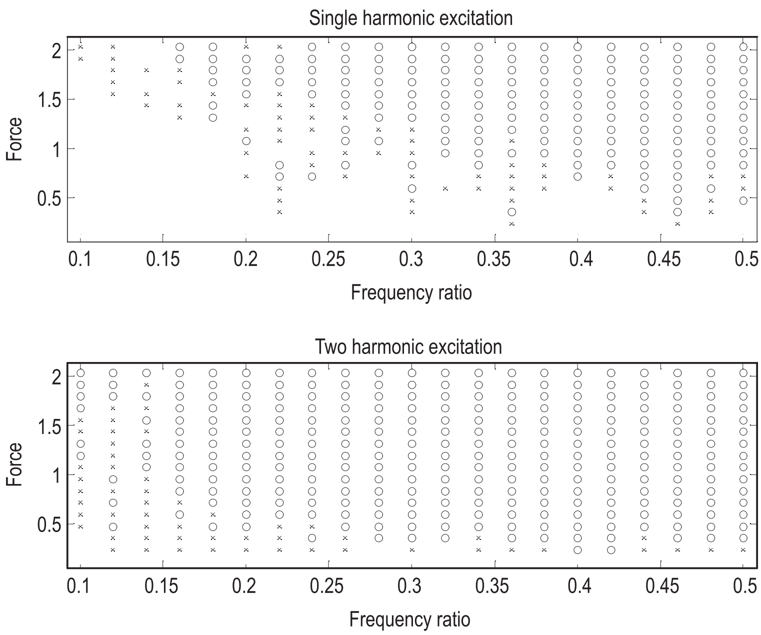

Fig. 2 Success over the excitation parameter space $F - \eta$, for single and two excitation cases. Symbol 'o' stands for $E_{par} \leq 0.01$ and 'x' for $0.01 < E_{par} \leq 0.05$. The blank spaces correspond to either $E_{par} > 0.05$ or the response not period-one type

Case 2: Corresponding two harmonic excitation amplitudes: $f_1 = F$, $f_2 = 0.2F$ and frequencies: $\Omega_2 = 2\Omega_1 = 2\Omega$. Identification tests were conducted for a set of parameters and a comparison for the success was made between a single and two harmonic excitations. The steady state responses, which are not $P1$ (period-one), are excluded from the analysis. Table 2 and the corresponding Fig. 2 gives the comparison result for tests conducted over a range of excitation parameters. There is a significant improvement in the success of identification while employing two-harmonic excitation in place of one-harmonic.

Figure 3 shows the success variation over a range of nonlinear stiffness α. The two harmonic input case shows improvement in the identification results. To examine the possible reason for the improvement, consider the CL matrix in the (25). The condition number of this matrix indicates the quality of inversion of the matrix. Hence the condition number is evaluated for the two cases. It appears that that a lower condition number gives a better result in many cases. However this is not always true and further investigations are needed.

30 equally spaced points over F-η grid is used at each value of α to obtain this comparison.

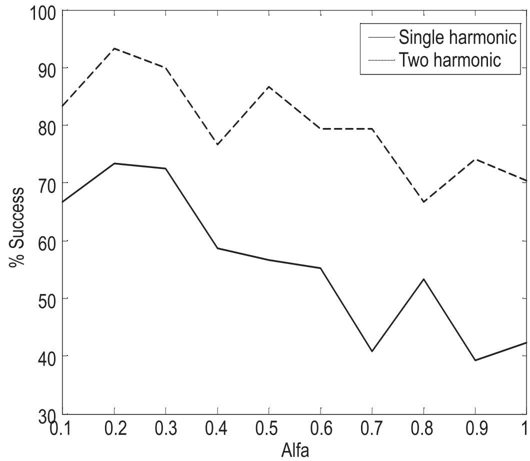

Fig. 3 Percentage success (for $E_{\text{par}} \leq 0.05$) variation with nonlinear stiffness α for $[mck_l] =$ [0.10.021]

5 Conclusion

Parametric system identification of a Duffing oscillator is carried out using harmonic balance method. In this work, an extended HB method, which uses a combination of two harmonic inputs, is suggested to overcome the problem of numerical difficulties encountered with a sinusoidal excitation. For the chosen parameters the two-harmonic excitation gives significant improvement over the single harmonic excitation. More tests over a wide range of both the system and excitation parameters are necessary to obtain a pattern and for generalization.

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